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Persistent current flux correlations calculated by quantum chaology

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Abstract. We consider classically chaotic systems with the topology of a ring threaded by quantum flux ϕ. Using semiclassical asymptotics, we calculate the flux-averaged autocorrelation function $C(\phi)$ of slopes of the energy levels (persistent currents), normalized by the mean level spacing, for flux values differing by ϕ. Our result furnishes the uniform approximation

$$C(\phi) \approx -\frac{\sin^2(\pi\phi) - 1/w^{*2}}{[\sin^2(\pi\phi) + 1/w^{*2}]^2}.$$

Here w^*, the RMS winding number of the classical periodic orbits whose period is connected by Heisenberg's relation to the mean level spacing, is a (large) semiclassical parameter, of order $1/\hbar^{(D-1)/2}$ for a system with D freedoms.

1. Introduction

Consider a charged quantum particle confined by a scalar potential to move in a ring threaded by quantum (Aharonov–Bohm) flux ϕ $(= (h/e) \times \text{flux})$. Then the energy levels E_n depend on ϕ, and the normalized derivatives

$$\bar{d}\frac{\mathrm{d}E_n}{\mathrm{d}\phi}(\phi) \tag{1}$$

where $\bar{d}$ is the mean level density (reciprocal of the mean level spacing), determine persistent currents, which are of particular interest in mesoscopic systems where the ring contains disorder in the form of many elastic scatterers. Szafer and Altshuler (1993) have introduced the autocorrelation function

$$C(\phi) = \bar{d}^2 \int_0^1 \mathrm{d}\phi_0 \left\langle \frac{\mathrm{d}E_n}{\mathrm{d}\phi}(\phi_0 + \phi)\frac{\mathrm{d}E_n}{\mathrm{d}\phi}(\phi_0)\right\rangle_n . \tag{2}$$

Here the averages are over a flux period $0 \leqslant \phi_0 < 1$ and levels n lying in an energy range ΔE which is classically small but quantally large (in the sense that it includes many levels). They speculate that this function 'offers a possibly universal quantum mechanical characterization of chaotic systems', independent of whether the chaos originates in mesoscopic disorder or deterministic instability of the classical trajectories. To support this view they note the agreement between an analytical calculation for ϕ not too small, based on averaging over mesoscopic disorder, and numerical calculations on classically chaotic ring-shaped billiards.

Our purpose here is to advance this argument with an analytical calculation of $C(\phi)$ over the whole range of ϕ, based on semiclassical asymptotics using the sum over periodic classical orbits of Gutzwiller (1971, 1990). Universality arises because the relevant sums are dominated by long orbits; this also occurs for other spectral statistics, and has led to the development of special techniques (Hannay and Ozorio de Almeida 1984, Berry 1985, 1991) which we shall also employ here. We consider the ballistic regime, where the mean free path for scattering is of the same order as the size of the ring; for example the ring could be a hollow 'billiard', the simplest planar case being a circular hole in a square box (Sinai's billiard). Therefore we cannot compare our results with the semiclassical theory developed for mesoscopic systems by Argaman *et al* (1993a), because they make essential use of the assumption that the classical motion is diffusive (mean free path much smaller than the ring); moreover, they calculate not the function $C(\phi)$ but the grand canonical average of $C(0)$.

Related semiclassical arguments have been applied by Serota (1992) to calculate the total magnetic moment of an Aharonov–Bohm billiard.

2. Autocorrelation in terms of the spectral staircase

Consider the smoothed spectral staircase (eigenvalue counting function)

$$N_\epsilon(E,\phi) = \sum_n \Theta_\epsilon(E - E_n(\phi)). \tag{3}$$

Here Θ_ϵ denotes the unit step, and the energy smoothing ϵ is smaller than the mean level spacing. The energy derivative of this staircase is the smoothed spectral density

$$d_\epsilon(E,\phi) = \frac{\partial N_\epsilon}{\partial E}(E,\phi) \tag{4}$$

whose energy or flux average is the mean level density $\bar{d}(E)$, which we henceforth denote simply by $\bar{d}$.

We will argue that the desired autocorrelation function $C(\phi)$ is semiclassically equal to an apparently very different quantity defined in terms of N_ϵ. This is

$$F(\phi,\epsilon) = \int_0^1 \mathrm{d}\phi_0 \left\langle \frac{\mathrm{d}N_\epsilon}{\mathrm{d}\phi}(E,\phi_0+\phi)\frac{\mathrm{d}N_\epsilon}{\mathrm{d}\phi}(E,\phi_0)\right\rangle_E \tag{5}$$

where the average is over flux ϕ and the same energy range ΔE as in (2), and ϵ takes a particular value, soon to be fixed, which depends on the kind of smoothing. Using (3) we obtain

$$F(\phi,\epsilon) = \int_0^1 \mathrm{d}\phi_0 \sum_m \sum_n \frac{\mathrm{d}E_m}{\mathrm{d}\phi}(\phi_0+\phi)\frac{\mathrm{d}E_n}{\mathrm{d}\phi}(\phi_0)\langle\delta_\epsilon(E - E_m(\phi_0+\phi))\delta_\epsilon(E - E_n(\phi_0))\rangle_E. \tag{6}$$

We choose to employ Lorentzian smoothing, for which the step is defined in terms of the smoothed delta function by

$$\delta_\epsilon(x) \equiv \frac{\mathrm{d}}{\mathrm{d}x}\Theta_\epsilon(x) \equiv -\frac{1}{\pi}\,\mathrm{Im}\,\frac{1}{x+\mathrm{i}\epsilon} = \frac{\epsilon}{\pi(x^2+\epsilon^2)} \tag{7}$$

and remark that we have also carried through the subsequent calculations with Gaussian smoothing and obtained the same final result (see (37)) for $C(\phi)$.

We relate F to C by a two-step argument. First, consider ϕ sufficiently large that the two delta functions in (6) are uncorrelated. Then we can replace each of their sums by $\bar{d}$. This gives

$$F(\phi,\epsilon) \approx \bar{d}^2 \int_0^1 \mathrm{d}\phi_0 \left\langle \frac{\mathrm{d}E_m}{\mathrm{d}\phi}(\phi_0+\phi)\frac{\mathrm{d}E_n}{\mathrm{d}\phi}(\phi_0)\right\rangle_{m,n} \tag{8}$$

where the average is over pairs of states in the range ΔE. As in Szafer and Altshuler (1993), we neglect the off-diagonal terms ($n \neq m$), arguing that for different states m and n the energy slopes will be only weakly correlated and their product will average to zero or will be semiclassically small. Thus, for sufficiently large ϕ, $F \approx C$, independently of ϵ.

The second step is to consider $\phi = 0$. For a non-degenerate spectrum, again only the diagonal ($n = m$) terms contribute to F. This allows us to use the result, which follows from (7), that

$$\delta_\epsilon^2(x) \approx \frac{1}{2\pi\epsilon}\delta_{\epsilon/2}(x) \tag{9}$$

for $\epsilon \ll \bar{d}^{-1}$. Thus

$$F(0,\epsilon) = \frac{1}{2\pi\epsilon}\int_0^1 \mathrm{d}\phi_0 \sum_n \langle\delta_{\epsilon/2}(E-E_n(\phi_0))\rangle_E \left[\frac{\mathrm{d}E_n}{\mathrm{d}\phi}(\phi_0)\right]^2 . \tag{10}$$

The energy average enables the sum over delta functions to be replaced by $\bar{d}$, giving

$$F(0,\epsilon) = \frac{\bar{d}}{2\epsilon\pi}\int_0^1 \mathrm{d}\phi_0 \left\langle\left[\frac{\mathrm{d}E_n}{\mathrm{d}\phi}(\phi_0)\right]^2\right\rangle_n \tag{11}$$

where the n-average is over states in the range ΔE. Comparison with (2) now shows that

$$C(0) = F\left(0, \epsilon = \frac{1}{2\pi\bar{d}}\right). \tag{12}$$

Taken together with the result for large ϕ, this strongly suggests that the choice

$$\epsilon = \frac{1}{2\pi\bar{d}} \tag{14}$$

in F provides a uniform approximation to C over the whole range of ϕ. We henceforth assume this (noting in passing that it will ultimately emerge that the limiting forms of C, as given by (38) and (39), are actually independent of ϵ), and now proceed to the semiclassical evaluation of $F(\phi,\epsilon)$ from the definition (5).

3. Semiclassical theory

For the staircase in (5) we employ the energy integral of Gutzwiller's formula for the spectral density. This involves the stabilities and actions of the classical periodic orbits at the energy E considered. Since the particles are shielded from the flux, this has no effect on the Newtonian trajectories, but does change the phase of the semiclassical contributions; of course, this is just the Aharonov–Bohm effect. If S is the action of a periodic orbit, the phase is changed as follows:

$$\frac{S}{\hbar} \to \frac{S}{\hbar} + 2\pi w\phi \tag{15}$$

where w is the number of times the orbit winds around the flux line. The effect of the Lorentzian smoothing (7) is to replace E by $E + \mathrm{i}\epsilon$. Thus, labelling closed orbits by j, we have

$$N_\epsilon(E,\phi) = \bar{N}(E) + \sum_j B_j(E)\exp\left\{\mathrm{i}\left[\frac{S_j(E)}{\hbar} + 2\pi w_j\phi\right]\right\}\exp\left\{-\frac{\epsilon T_j(E)}{\hbar}\right\}. \tag{16}$$

Here the sum is over both positive and negative traversals, $\bar{N}$ is the mean ('Weyl') staircase, which, to leading asymptotic order, is independent of flux, T_j is the period of the orbit, and

$$B_j = \frac{\exp\{\mathrm{i}\mu_j\}}{2\pi\sqrt{\det(M_j - 1)}} \tag{17}$$

where M_j denotes the linearized Poincaré return map of the orbit, and μ_j the Maslov phase.

Equation (5) involves bilinear staircase products and so contains double sums over periodic orbits. Flux averaging eliminates all pairs of orbits except those with the same winding number. Thus

$$F(\phi,\epsilon) = 4\pi^2\left\langle\sum_j\sum_k |B_jB_k| w_j^2 \exp\left\{-\frac{\epsilon(T_j+T_k)}{\hbar}\right\} \times \exp\left\{\mathrm{i}\left(\frac{S_j - S_k}{\hbar} + 2\pi w_j\phi\right)\right\}\delta_{w_j,w_k}\right\rangle_E. \tag{18}$$

We separate the diagonal and off-diagonal contributions, and write

$$F(\phi,\epsilon) = F_{\mathrm{diag}}(\phi,\epsilon) + F_{\mathrm{off}}(\phi,\epsilon). \tag{19}$$

In section 5 we shall show that F_{off} is negligible.

For the diagonal contribution we have

$$F_{\mathrm{diag}}(\phi,\epsilon) = 4\pi^2\sum_j |B_j|^2 w_j^2\cos(2\pi w_j\phi)\exp\left\{-\frac{2\epsilon T_j}{\hbar}\right\}. \tag{20}$$

Note that there is not the customary factor 2 from the coherent interference of each orbit with its geometrically identical time-reverse, which might be though to contribute here because the Newtonian trajectories possess time-reversal symmetry, and which is a principal source of the difference between statistics of the Gaussian unitary ensemble (GUE) and Gaussian orthogonal ensemble in random-matrix theory. The reason for its non-appearance here is that although the orbit and its time-reverse have the same actions, they have opposite winding numbers, so the contribution of the pair vanishes in the average over flux (cf the Kronecker delta in (18)), reflecting the fact that for non-zero flux the quantum (and semiclassical) dynamics lacks time-reversal symmetry.

To evaluate the sum (20), we order the orbits by their period T_j. Orbits proliferate exponentially, but the contributions $|B_j|^2$ are exponentially damped. The near-balancing of these effects is the essence of the classical sum rule of Hannay and Ozorio de Almeida (1984), and enables us to make the replacement

$$\sum_j |B_j|^2 \cdots \to \frac{1}{2\pi^2}\int_0^\infty \frac{\mathrm{d}T}{T}\cdots. \tag{21}$$

(A quick way to see the truth of this result is to anticipate that in our application the integrand will be even, and note that the density of the distribution of periods over long orbits is $\exp\{\lambda|T|\}/|T|$, while the amplitudes (17) have the asymptotic form $\exp\{-\lambda|T|/2\}/2\pi$, where λ is the entropy of the classical motion.) The lower limit of zero is appropriate if the resulting integral converges, as it will.

4. Winding number average

Before making use of (21) to convert the sum (20) into an integral, we note that the winding numbers of the orbits in any small range of period will be irregularly distributed. Their distribution will be symmetric about zero, and it is natural to approximate it as Gaussian (see Berry and Robnik 1986, especially the appendix),

$$P(w) = \frac{\exp\{-w^2/2\langle w^2(T)\rangle\}}{\sqrt{2\pi\langle w^2(T)\rangle}} \tag{22}$$

with a variance increasing linearly with T, i.e.

$$\langle w^2(T)\rangle = \frac{\alpha T}{T_0} \tag{23}$$

where T_0 is the period of the shortest orbit, and α is a system-dependent dimensionless constant. (In writing (22) we have ignored a normalization constant differing from unity by $\mathrm{O}(\exp\{-2\pi^2\langle w^2(T)\rangle\})$, which is negligible in the following.)

The average in the diagonal contribution (20) can be evaluated by the Poisson sum formula:

$$\sum_{w=-\infty}^{\infty} P(w)w^2\cos(2\pi w\phi)$$
$$= \sum_{n=-\infty}^{\infty}\langle w^2(T)\rangle[1-4\pi^2(\phi-n)^2\overline{w^2(T)}]\exp\{-2\pi^2(\phi-n)^2\overline{w^2(T)}\}. \tag{24}$$

Thus on using (20) and (21) F_{diag} becomes

$$F_{\mathrm{diag}}(\phi,\epsilon) = \frac{2\alpha}{T_0}\sum_{n=-\infty}^{\infty}\int_0^\infty \mathrm{d}T\left(1-\frac{[2\pi(\phi-n)]^2\alpha T}{T_0}\right)$$
$$\times\exp\left\{-2\left(\frac{\epsilon}{\hbar}+\frac{\alpha}{T_0}[\pi(\phi-n)]^2\right)T\right\}. \tag{25}$$

The integral is elementary, and gives

$$F_{\rm diag}(\phi,\epsilon) = -\frac{1}{\pi^2}\sum_{n=-\infty}^{\infty}\frac{(\phi-n)^2-1/\pi^2 w_\epsilon^2}{[(\phi-n)^2+1/\pi^2 w_\epsilon^2]^2} \tag{26}$$

where

$$w_\epsilon \equiv \sqrt{\frac{\alpha\hbar}{\epsilon T_0}}. \tag{27}$$

A second application of Poisson summation now leads to

$$F_{\rm diag}(\phi,\epsilon) = -4\exp\left\{-\frac{2}{w_\epsilon}\right\}\frac{2(1+\exp\{-4/w_\epsilon\})\sin^2(\pi\phi)-(1-\exp\{-2/w_\epsilon\})^2}{[4\sin^2(\pi\phi)\exp\{-2/w_\epsilon\}+(1-\exp\{-2/w_\epsilon\})^2]^2}. \tag{28}$$

When $w_\epsilon \gg 1$ (as it will be in the following), this can, as illustrated in figure 1, be replaced by the approximation

$$F_{\rm diag}(\phi,\epsilon) \approx -\frac{\sin^2(\pi\phi)-1/w_\epsilon^2}{[\sin^2(\pi\phi)+1/w_\epsilon^2]^2}. \tag{29}$$

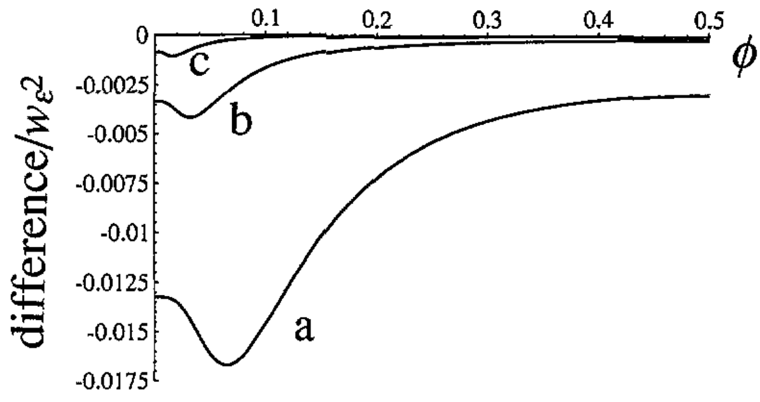

Figure 1. Difference between the right-hand sides of (28) and (29), normalized by w_ϵ^2, for (*a*) $w_\epsilon = 5$, (*b*) $w_\epsilon = 10$ and (*c*) $w_\epsilon = 20$.

For the autocorrelation we require F for the value of ϵ given by (14). The corresponding winding number is

$$w_{\epsilon=1/2\pi\bar{d}} = \sqrt{\frac{2\pi\alpha\hbar\bar{d}}{T_0}} \equiv w^*. \tag{30}$$

This quantity w^* has physical significance: it is the typical winding number of the orbits whose period $T^* = 2\pi\hbar\bar{d}$ is related by Heisenberg's principle to the mean level spacing (T^* plays an important role in the theory of spectral statistics). In a system with D freedoms ($D \geqslant 2$), the mean spacing is of order $\hbar^D$, so that w^* is of order $1/\hbar^{(D-1)/2}$ and therefore semiclassically large.

5. Estimate of $F_{\rm off}(\phi, \epsilon)$

Assuming no correlations between the winding numbers of different orbits, the winding number average in the off-diagonal part of (18) is (cf (22) and (24))

$$\sum_{w_1}\sum_{w_2} P(w_1)P(w_2)w_1w_2\exp\{i2\pi w_1\phi\}\delta_{w_1,w_2} = \sum_{w} P^2(w)w^2\exp\{i2\pi w_1\phi\}$$
$$= \frac{1}{4}\sqrt{\frac{\langle w^2\rangle}{\pi}}(1-2\pi^2\langle w^2\rangle\phi^2)\exp\{-\pi^2\langle w^2\rangle\phi^2\} \quad (31)$$

(here we have assumed $0 \leqslant \phi < \frac{1}{2}$ and neglected some exponentially small terms). Thus we obtain

$$F_{\rm off}(\phi,\epsilon) = \pi\sqrt{\frac{\alpha}{T_0}}\left\langle\sum\sum_{j\neq k}|B_jB_k|\exp\left\{-\frac{\epsilon(T_j+T_k)}{\hbar}\right\}\exp\left\{i\left(\frac{S_j-S_k}{\hbar}\right)\right\}\right.$$
$$\left.\times\sqrt{\pi T_j}\left(1-\frac{2\alpha T_j\pi^2\phi^2}{T_0}\right)\exp\left\{-\frac{\alpha T_j\pi^2\phi^2}{T_0}\right\}\right\rangle_E . \quad (32)$$

This can be expressed as an integral of the off-diagonal part of the *spectral form factor*, whose semiclassical expression (Berry 1985, 1991) is

$$K_{\rm off}\left(\frac{T}{2\pi\hbar\bar{d}}\right) = \frac{2\pi}{\hbar\bar{d}}\left\langle\sum\sum_{j\neq k}|B_jB_k|T_jT_k\exp\left\{i\left(\frac{S_j-S_k}{\hbar}\right)\right\}\delta\left[T-\tfrac{1}{2}(T_j+T_k)\right]\right\rangle_E . \quad (33)$$

The formula is

$$F_{\rm off}(\phi,\epsilon) = \sqrt{\frac{\alpha\hbar\bar{d}}{2T_0}}\int_0^\infty\frac{d\tau}{\tau^{3/2}}K_{\rm off}(\tau)\left(1-\frac{4\pi^3\alpha\hbar\bar{d}\phi^2\tau}{T_0}\right)$$
$$\times\exp\left\{-\tau\left(4\pi\epsilon\bar{d}+\frac{2\pi^3\alpha\hbar\bar{d}\phi^2}{T_0}\right)\right\} \quad (34)$$

(which is equivalent to the result of substituting the action correlation function of Argaman *et al* (1993b) directly into (32)).

For $K_{\rm off}$ we assume GUE statistics for the close correlations of the energy levels and employ the sum rule of Hannay and Ozorio de Almeida (1984) as in Berry (1985), that is,

$$K_{\rm off}(\tau) = (1-\tau)\Theta(\tau-1). \quad (35)$$

The lower limit of the integral (34) is now unity, and $F_{\rm off}$ can be estimated by replacing $\tau^{3/2}$ by unity. The result, written for the value of ϵ given by (14), is

$$F_{\rm off}\left(\phi,\epsilon=\frac{1}{2\pi\bar{d}}\right) \approx -\frac{w^*}{2\sqrt{\pi}}\frac{\exp\{-[2+(\pi w^*\phi)^2]\}}{[2+(\pi w^*\phi)^2]^3}[2-7(\pi w^*\phi)^2-2(\pi w^*\phi)^4] \quad (36)$$

where w^* is the winding number (30). Calculations (illustrated in figure 2) show that this function is negligible compared with $F_{\rm diag}$ (given by (29) with $w_\epsilon = w^*$) over the whole range of ϕ.

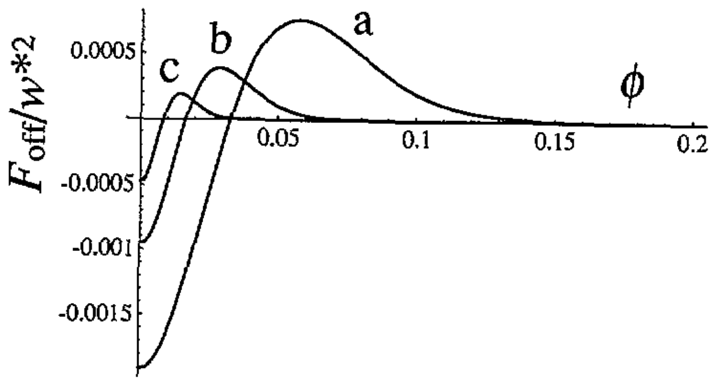

Figure 2. Off-diagonal contribution $F_{\rm off}(\phi, \epsilon = 1/2\pi\bar{d})$ (see (36)), normalized by w^{*2}, for (*a*) $w^* = 5$, (*b*) $w^* = 10$ and (*c*) $w^* = 20$.

6. Flux autocorrelation formula

From the arguments of section 2 showing that the desired function $C(\phi)$, defined by (2), can be identified with the staircase correlation $F(\phi, \epsilon = 1/2\pi\bar{d})$, and the calculations of sections 4 and 5 showing that F is semiclassically dominated by its diagonal part (29), we obtain, for the flux correlation of the persistent currents,

$$C(\phi) \approx -\frac{\sin^2(\pi\phi) - 1/w^{*2}}{[\sin^2(\pi\phi) + 1/w^{*2}]^2}. \tag{37}$$

Here the large semiclassical parameter w^* (of order $1/\hbar^{(D-1)/2}$) is the winding number (30). This remarkably simple formula is our main result. Curves of $C(\phi)$ for several values of w^* are shown in figure 3. (For values of the parameters such that w^* cannot be considered large, the full semiclassical approximation (28) should be used.)

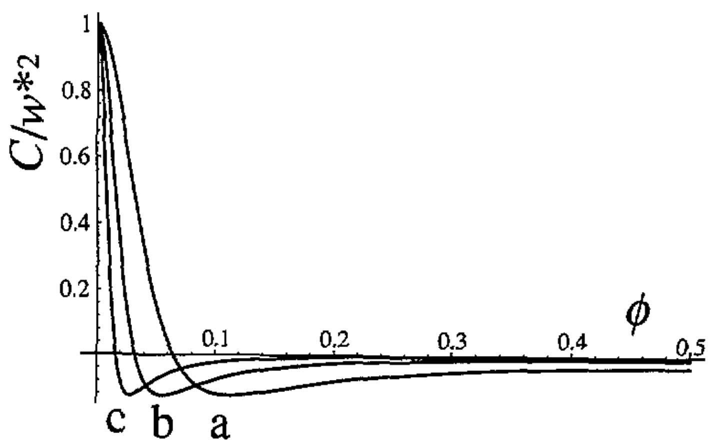

Figure 3. Persistent current flux correlations (see (37)), normalized by w^{*2}, for (*a*) $w^* = 5$, (*b*) $w^* = 10$ and (*c*) $w^* = 20$.

The periodic function $C(\phi)$ is large and positive for very small ϕ, and has the limiting value

$$C(0) = w^{*2} = \frac{2\pi\hbar\bar{d}\alpha}{T_0} \approx \frac{1}{\hbar^{D-1}}. \tag{38}$$

This accords with the elementary estimate obtained by considering $E_n(\phi)$ to fluctuate over approximately a mean level spacing $1/\bar{d}$ in a ϕ range of $1/w^*$, which according to the Gutzwiller formula (16) generates the spectral oscillations on the smallest energy scales.

The autocorrelation rapidly decreases, and falls through zero as ϕ increases through the small value $1/(\pi w^*)$. Thereafter ϕ is negative. It passes through a minimum which for large w^* is at $\sqrt{3}/(\pi w^*)$ and has a value $-w^{*2}/8$. For larger ϕ we reach the universal asymptotic form

$$C(\phi) \approx -\frac{1}{\pi^2\phi^2} \qquad \left(\frac{1}{\pi w^*} \ll \phi \ll \frac{1}{2}\right). \tag{39}$$

This is identical with the result obtained by Szafer and Altshuler (1993) for mesoscopic systems.

Finally, it is worth noting that the limiting forms (38) and (39) are actually independent of the choice (14) for ϵ, provided $\epsilon < \bar{d}^{-1}$. In the second case ($\phi \gg 1/\pi w^*$) this is immediately apparent since the corresponding limit of (29) is independent of w_ϵ. Essentially, it is due to the fact that the second term in the exponential in (25) dominates the convergence of the integral when ϕ is sufficiently large. In the first case, when $\phi = 0$, it would appear on first sight that the value of C should depend on (14), because it is related to w^*. That in fact it does not follows from the expression

$$C(0) = 2\pi\bar{d} \lim_{\epsilon\to 0} \epsilon F(0, \epsilon) \tag{40}$$

which is a consequence of (11). Hence, from (25),

$$C(0) = \frac{4\pi\alpha\bar{d}}{T_0} \lim_{\epsilon\to 0} \epsilon \int_0^\infty \mathrm{d}T \, \exp\left\{-\frac{2\epsilon T}{\hbar}\right\}. \tag{41}$$

The limit is clearly independent of ϵ and leads again to (38), as it must to maintain consistency with the derivation of (40) from (11)—a derivation which is itself independent of ϵ provided $\epsilon \ll \bar{d}^{-1}$, because of (9). Hence the choice (14) could affect only the form of the interpolation between the limits (38) and (39).

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References

Argaman N, Imry Y and Smilansky U 1993a *Phys. Rev.* B **47** 4440–57

Argaman N, Dittes F-M, Doron E, Keating J P, Kitaev A Yu, Sieber M and Smilansky U 1993b *Phys. Rev. Lett.* **71** 4326–9

Berry M V 1985 *Proc. R. Soc.* A **400** 229–51

—— 1991 Some quantum-to-classical asymptotics *Les Houches Lecture Series LIV (1989)* ed M-J Giannoni *et al* (Amsterdam: North-Holland) pp 251–304

Berry M V and Robnik M 1986 *J. Phys. A: Math. Gen.* **19** 649–68

Gutzwiller M C 1971 *J. Math. Phys.* **12** 343–58

—— 1990 *Chaos in Classical Quantum Mechanics* (New York: Springer)

Hannay J H and Ozorio de Almeida A M 1984 *J. Phys. A: Math. Gen.* **17** 3429–40

Serota R A 1992 *Solid State Commun.* **84** 843–5

Szafer A and Altshuler B L 1993 *Phys. Rev. Lett.* **70** 587–90